

For this question, you may use the formulae for $\frac{d}{dx} \sinh x$, $\frac{d}{dx} \cosh x$ and/or $\frac{d}{dx} \tanh x$ without proving them. SCORE: ____ / 8 PTS

If you need to use the formula for the derivative of any other hyperbolic function, you must prove it.

- [a] Without using the exponential formula for $\operatorname{sech} x$, prove the formula for $\frac{d}{dx} \operatorname{sech} x$.

$$\begin{aligned} \frac{d}{dx} \frac{1}{\cosh x} &= -(\cosh x)^{-2} \sinh x \quad (2) \\ &= -\frac{1}{\cosh x} \frac{\sinh x}{\cosh x} \\ &= -\operatorname{sech} x \tanh x \quad (1) \end{aligned}$$

- [b] Without using the logarithmic formula for $\tanh^{-1} x$, prove the formula for $\frac{d}{dx} \tanh^{-1} x$.

$$\begin{aligned} y &= \tanh^{-1} x \\ x &= \tanh y \quad (1) \\ 1 &= \operatorname{sech}^2 y \frac{dy}{dx} \quad (1\frac{1}{2}) \\ 1 &= (1 - \tanh^2 y) \frac{dy}{dx} \\ 1 &= (1 - x^2) \frac{dy}{dx} \end{aligned}$$

$$\begin{aligned} \cosh^2 y - \sinh^2 y &= 1 \\ (1) \quad 1 - \tanh^2 y &= \operatorname{sech}^2 y \end{aligned}$$

$$\frac{dy}{dx} = \frac{1}{1 - x^2} \quad (1\frac{1}{2})$$

Find $\lim_{x \rightarrow \infty} \coth x$ algebraically.

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$$\underbrace{\lim_{x \rightarrow \infty} \frac{e^{2x} + 1}{e^{2x} - 1}}_{\textcircled{1} \quad \frac{\infty}{\infty}} = \underbrace{\lim_{x \rightarrow \infty} \frac{2e^{2x}}{2e^{2x}}}_{\text{L'HÔPITAL'S RULE } \textcircled{1}} = \underbrace{1}_{\textcircled{1}}$$

If $\coth x = -5$, find $\sinh x$.

ALTERNATE SOLUTION ON
NEXT PAGE

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$$\cosh^2 x - \sinh^2 x = 1$$

$$\coth^2 x - 1 = \operatorname{csch}^2 x \quad (1)$$

$$24 = \operatorname{csch}^2 x \quad (1)$$

$$\operatorname{csch} x = \neq 2\sqrt{6} \rightarrow$$
$$= -2\sqrt{6} \quad (\frac{1}{2})$$

$$\sinh x = -\frac{1}{2\sqrt{6}} = -\frac{\sqrt{6}}{12} \quad (\frac{1}{2})$$

$$\coth x = \frac{\cosh x}{\sinh x}$$

SINCE $\coth x < 0$

AND $\cosh x > 0$ (FOR ALL x),

$\sinh x < 0$

SO $\operatorname{csch} x < 0$

(1/2)

(1/2)

If $\coth x = -5$, find $\sinh x$.

$\left(\frac{1}{2}\right)$ POINT
EACH

$$\tanh x = -\frac{1}{5}$$

$$\frac{\sinh x}{\cosh x} = -\frac{1}{5}$$

$$\sinh x = -\frac{1}{5} \cosh x$$

$$= -\frac{1}{2\sqrt{6}} = -\frac{\sqrt{6}}{12}$$

SCORE: ____ / 4 PTS

$$\cosh^2 x - \sinh^2 x = 1$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\frac{24}{25} = \operatorname{sech}^2 x$$

$$\frac{2\sqrt{6}}{5} = \operatorname{sech} x > 0$$

$$\frac{5}{2\sqrt{6}} = \cosh x$$

Prove the logarithmic formula for $\sinh^{-1} x$ given in your textbook.

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NOTE: This is NOT a question about derivatives.

$$y = \sinh^{-1} x$$

$$\textcircled{1} \quad x = \sinh y = \frac{e^y - e^{-y}}{2}$$

$$\textcircled{\frac{1}{2}} \quad x = \frac{z - \frac{1}{z}}{2}$$

$$2x = z - \frac{1}{z}$$

$$2xz = z^2 - 1$$

$$\textcircled{\frac{1}{2}} \quad 0 = z^2 - 2xz - 1$$

$$\textcircled{1} \quad z = \frac{2x \pm \sqrt{4x^2 + 4}}{2} = x \pm \sqrt{x^2 + 1}$$

$$\text{LET } z = e^y, \text{ SO } z > 0 \quad \textcircled{\frac{1}{2}}$$



$$e^y = z = x + \sqrt{x^2 + 1} \quad \textcircled{\frac{1}{2}}$$

$$\sinh^{-1} x = y = \ln(x + \sqrt{x^2 + 1}) \quad \textcircled{\frac{1}{2}}$$

Find $\frac{d}{dx} x^2 \cosh^{-1}(x^5)$. Simplify your final answer as a single fraction.

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You may use the derivatives of any hyperbolic or inverse hyperbolic functions from your textbook without proving them.

$$\begin{aligned} & 2x \cosh^{-1} x^5 + x^2 \frac{5x^4}{\sqrt{(x^5)^2 - 1}} \\ = & \underbrace{2x \cosh^{-1} x^5}_{\textcircled{1}} + \frac{\underbrace{5x^6}_{\textcircled{1}}}{\underbrace{\sqrt{x^{10} - 1}}_{\textcircled{2}}} \end{aligned}$$